Universal Reinforcement Learning

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DeepMind

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Abstract

Title: Universal Artificial Intelligence: Practical Agents and Fundamental Challenges

Abstract: Foundational theories have contributed greatly to the scientific progress in many fields. Examples include ZFC in mathematics and universal Turing machines in computer science. Universal Artificial Intelligence (UAI) is an increasingly well-studied foundational theory for artificial intelligence. It is based on ancient principles in the philosophy of science and modern developments in information and probability theory.

The main focus of this tutorial will be on an accessible explanation of the UAI theory and AIXI, and on discussing three approaches to approximating it effectively. UAI also enables us to reason precisely about the behaviour of yet-to-be-built future AIs, and gives us a deeper appreciation of some fundamental problems in creating intelligence.

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Blocks

Block 1: UAI Basics

2pm – 2:45pm

Block 2: Hands-on Examples

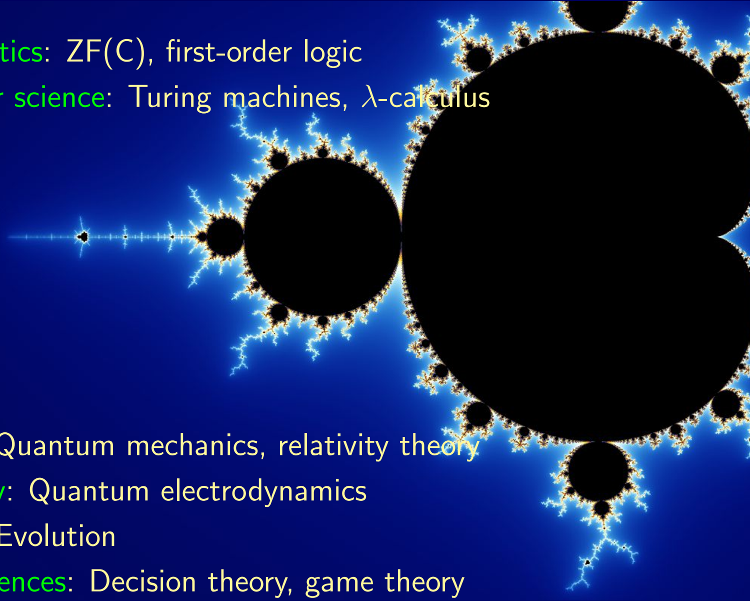
3pm – 4:30pm (coffee break in the middle)

Block 3: AI Safety

4:45pm – 5:45pm

Foundational Theories

- Mathematics: ZF(C), first-order logic
- Computer science: Turing machines, λ-calculus



- Physics: Quantum mechanics, relativity theory
- Chemistry: Quantum electrodynamics
- Biology: Evolution
- Social sciences: Decision theory, game theory

Theories of Intelligence

- Cognitive psychology
- Behaviourism
- Philosophy of mind
- Neuroscience
- Linguistics
- Anthropology
- Machine Learning
- Logic
- Computer science
- Biological evolution
- Economics

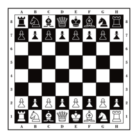
	Thinking	**Acting**
humanly	Cognitive science	Turing test, behaviourism
rationally	Laws of thought	AI: Doing the right thing

Approaches to AI

	Deduction centered	Induction centered
Main technique	Logic/symbolic reasoning	Prob. theory, ML
Agent goal	Logical specification	Reward (RL)
Noise/uncertainty	Brittle	Robust
Grounding	Problem	In reward
Foundational theory	No	UAI
Right thing to do		

Example

Medical expert systems, chess playing agents



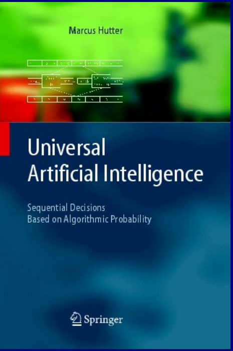
Example

AlphaGo, DQN, self-driving cars

nature
LEARNING CURVE
Self-taught AI software attains human-level performance in video games

What is the right thing to do?

Universal Artificial Intelligence (UAI)



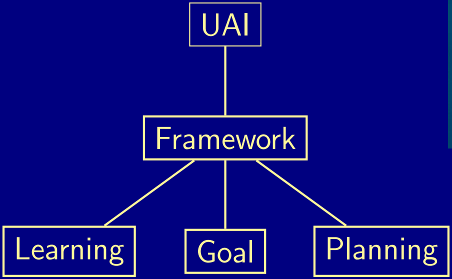
Marcus Hutter

Universal Artificial Intelligence

Sequential Decisions Based on Algorithmic Probability

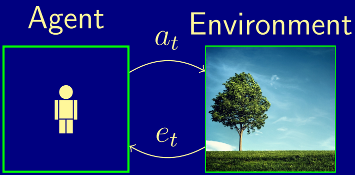
Springer

A foundational theory of AI



Answers: **What is the right thing to do?**

Framework

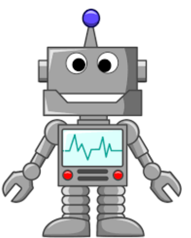


At each time step t, the agent

- submits action a_t
- receives percept e_t

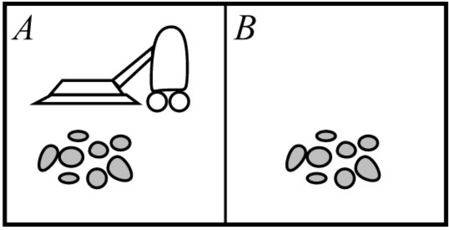
History $æ_{<t} = a_1 e_1 a_2 e_2 \dots a_{t-1} e_{t-1}$

Set of histories: $(\mathcal{A} \times \mathcal{E})^*$



Examples

Vacuum cleaner world



$\mathcal{E} = \{\text{dirt}, \text{no dirt}\}$
$\mathcal{A} = \{\text{suck}, \text{move left}, \text{move right}\}$

Stock trading

$\mathcal{E} = \mathbb{R}^+$ (price of stock)
$\mathcal{A} = \{\text{buy}, \text{sell}\}$

Agent and Environment

Agent

Policy

$\pi : (\mathcal{A} \times \mathcal{E})^* \to \mathcal{A}$

Next action

$a_t = \pi(æ_{<t})$

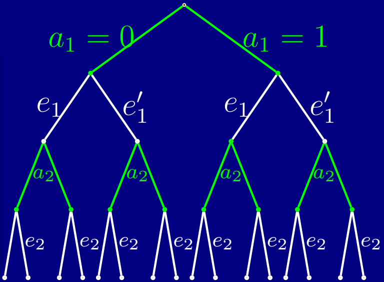
Environment

Distribution

$\mu : (\mathcal{A} \times \mathcal{E})^* \times \mathcal{A} \rightsquigarrow \mathcal{E}$

Probability of next percept:

$\mu(e_t \mid æ_{<t} a_t)$



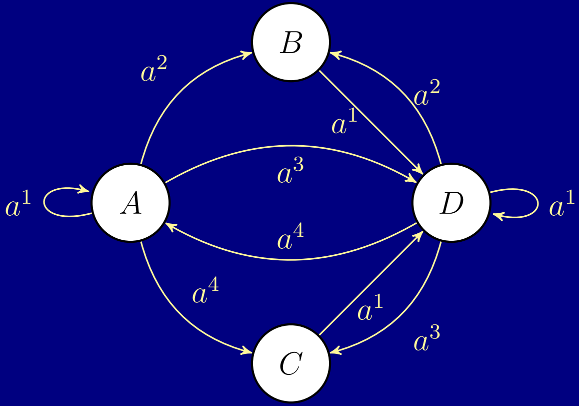
$a_1 = \pi(\epsilon)$

$e_1 \sim \mu(\cdot \mid a_1)$

$a_2 = \pi(a_1 e_1)$

$e_2 \sim \mu(\cdot \mid a_1 e_1 a_2)$

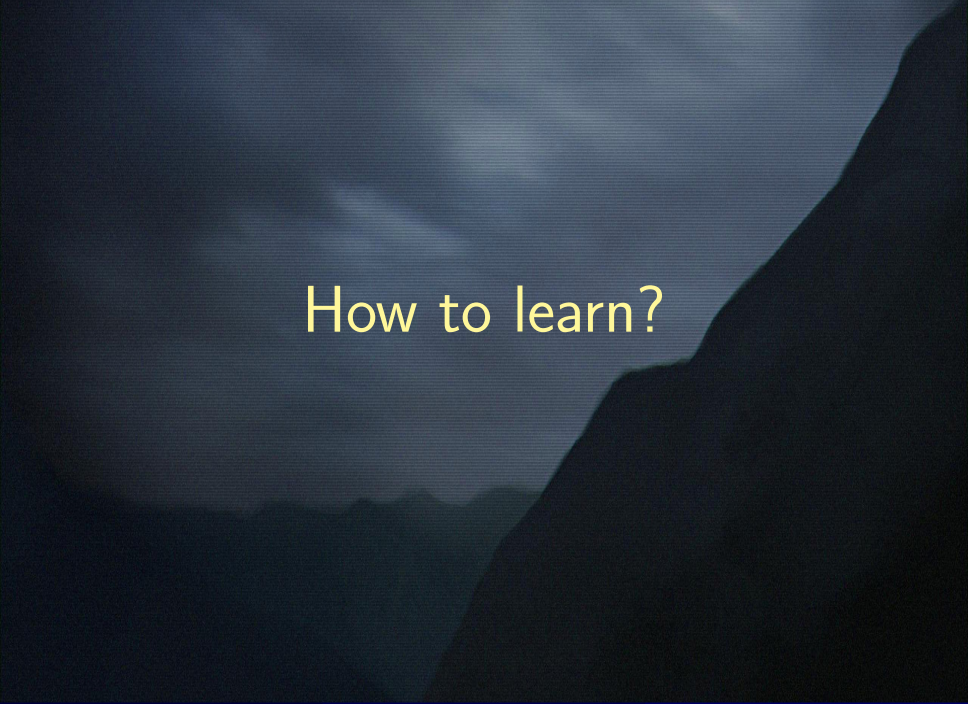
Markov Decision Process (MDP)



Environment $(s, a) \mapsto (s', r')$
Policy $\pi : \mathcal{S} \to \mathcal{A}$

Histories vs. States

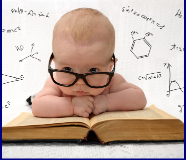
	History (UAI)	State (MDP)
Percept	e	(s, r)
Hidden states	Yes	POMDP
Infinite no. states	Yes	Normally not
Non-stationary env.	Yes	Can be added
Agents/algorithms	Policy	Sequence of policies
Learning	Harder	MDP: Easy in principle



How to learn?

Induction

Learn



$\Longrightarrow$

Predict

$\Longrightarrow$

Plan

$\Longrightarrow$

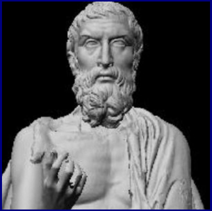
Act

True environment μ unknown

Principles

Occam (1285–1347)

Prefer the simplest consistent hypothesis



Epicurus (341–270 BC)

Keep all consistent hypotheses

Bayes (1701–1761)

$$\Pr(\mathrm{Hyp} \mid \mathrm{Data}) = \frac{\Pr(\mathrm{Hyp}) \Pr(\mathrm{Data} \mid \mathrm{Hyp})}{\sum_{H_i \in \mathcal{H}} \Pr(H_i) \Pr(\mathrm{Data} \mid H_i)}$$

Principles

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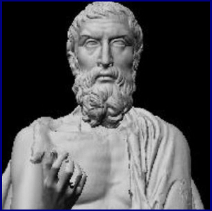
Prefer the simplest consistent hypothesis



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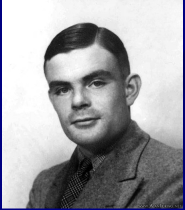
Remaining Questions

Hypothesis class $\mathcal{H}$:
What environments μ can agent can possibly encounter?

Prior:
What is the prior probability $\Pr(\mu)$ for each environment $\mu \in \mathcal{H}$?



Turing (1912–1954)

"It is possible to invent a single machine which can be used to compute any computable sequence."

Solomonoff Induction

Use computer programs p as hypotheses/environments

Given Turing-complete programming language U, programs can

- describe essentially any environment
- be checked for consistency: is $p(a_{<t}) = e_{<t}$?
- be used for prediction: compute $p(a_{<t}a_t)$
- be ranked by simplicity: $\Pr(p) = 2^{-\ell(p)}$

Solomonoff (1926–2009)

Make a weighted prediction based on all consistent programs, with short programs weighted higher

Solomonoff-Hutter's Universal Distribution

$$M(e_{<t} \mid a_{<t}) = \sum_{p\,:\,p(a_{<t})=e_{<t}} 2^{-\ell(p)}$$

where

- $a_{<t}$ action sequence
- $e_{<t}$ percept sequence
- p computer program
- $\ell(p)$ length of p

Predict with

$$M(e_t \mid æ_{<t} a_t) = \frac{M(e_{<t} e_t \mid a_{<t} a_t)}{M(e_{<t} \mid a_{<t})}$$

Solomonoff-Hutter's Universal Distribution

$$M(e_{<t} \mid a_{<t}) = \sum_{p \,:\, p(a_{<t}) = e_{<t}} 2^{-\ell(p)}$$

- $a_{<t}$ action sequence
- $e_{<t}$ percept sequence
- p computer program
- $\ell(p)$ length of p

- Occam: Simpler program higher weight
- Epicurus: All consistent programs
- Bayes: Discard inconsistent programs
- Turing: Any computable environment

Predict with

$$M(e_t \mid \text{æ}_{<t} a_t) = \frac{M(e_{<t} e_t \mid a_{<t} a_t)}{M(e_{<t} \mid a_{<t})}$$

Examples

$$M(e_{<t} \mid a_{<t}) = \sum_{p\,:\,p(a_{<t})=e_{<t}} 2^{-\ell(p)}$$

$M(010101010101 \mid 010101010101) =$ high

short program (low $\ell(p)$):

```
procedure $\textsc{MirrorEnvironment}$
    while true do:
        $x \leftarrow$ action input
        output percept $\leftarrow x$
    end while
end procedure
```

$M(011001110110 \mid 000000000000) =$ low

program must encode 011001110110 (high $\ell(p)$)

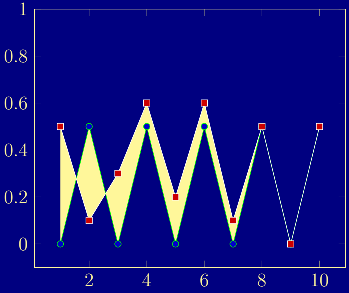
Results Solomonoff Induction

Theorem (Prediction error)

For any computable environment μ and any actions $a_{1:\infty}$:

$$\sum_{t=1}^{\infty} \mathbb{E}_\mu \Big[\underbrace{M(0 \mid æ_{<t}a_t) - \mu(0 \mid æ_{<t}a_t)}_{\textit{prediction error at time } t} \Big]^2 \overset{+}{\leq} \frac{1}{2} \ln 2 \cdot K(\mu)$$

- Solomonoff induction only makes finitely many prediction errors
- The environment μ may be deterministic or stochastic



Agent can learn any computable environment

What is the purpose?

Goal = reward

What should be the goal of the agent?

Assumption

$e = (o, r)$, where

- o observation
- $r \in [0, 1]$ reward

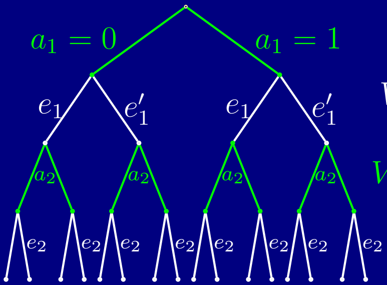
The goal is to maximise return = "discounted sum of rewards"

$$\sum_{k=1}^{\infty} \gamma^k r_k = \gamma r_1 + \gamma^2 r_2 + \gamma^3 r_3 + \dots$$

Expected Performance

The *expected return* is called value:

$$V_\mu^\pi(æ_{<t}) = \mathbb{E}_\mu^\pi\left[\sum_{k=1}^{\infty}\gamma^k r_k \;\middle|\; æ_{<t}\right]$$



$$V_\mu^\pi(\epsilon) = V_\mu^\pi(a_1) \text{ with } a_1 = \pi(\epsilon)$$

$$V_\mu^\pi(a_1) = \sum_{e_1}\mu(e_1 \mid a_1)\big[r_1 + \gamma V_\mu^\pi(a_1e_1)\big]$$

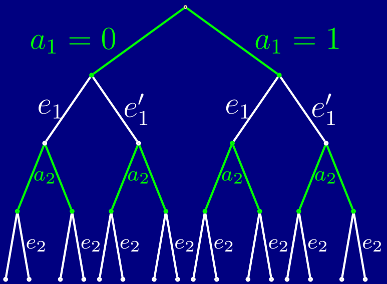
$$V_\mu^\pi(a_1e_1) = V_\mu^\pi(a_1e_1a_2) \text{ with } a_2 = \pi(a_1e_1)$$

$$V_\mu^\pi(a_1e_1a_2) = \\ \textstyle\sum_{e_2}\mu(e_2 \mid a_1e_1a_2)\big[r_2 + \gamma V_\mu^\pi(æ_{1:2})\big]$$

Expectimax Planning

The *expected return* is called value: $V^\pi_\mu(æ_{<t}) = \mathbb{E}^\pi_\mu[R(æ_{1:\infty}) \mid æ_{<t}]$

$$\sum_{k=1}^{\infty} \gamma^k r_k = \underbrace{r_1 + \gamma r_2 + \cdots \gamma^{m-1} r_m}_{\text{effective horizon}} + \underbrace{\gamma^m r_{m+1} + \cdots}_{<\epsilon} \approx \sum_{k=1}^{m} \gamma^k r_k$$



Optimal policy:
$\pi^* = \arg\max_\pi V^\pi_\mu$

An ϵ-optimal policy can be found in any environment μ

$$a_1^* = \arg\max_{a_1} \sum_{e_1} \mu(e_1 \mid a_1) \max_{a_2} \sum_{e_2} \mu(e_2 \mid a_1 e_1 a_2) \ldots \max_{a_m} \sum_{e_m} \mu(e_m \mid æ_{<m} a_m) R(æ_{1:m})$$

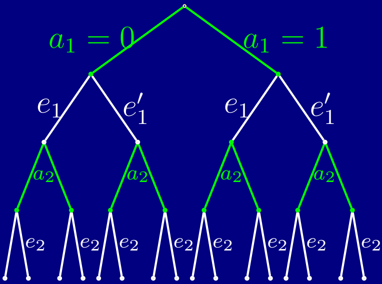
The right thing to do is. . .

Expectimax in Unknown Environments: AIXI

AIXI replaces μ with M: $\pi_{\text{AIXI}} = \arg\max_{\pi} V_M^{\pi}$

$$a_1^* = \arg\max_{a_1} \sum_{e_1} M(e_1 \mid a_1) \max_{a_2} \sum_{e_2} M(e_2 \mid a_1 e_1 a_2) \dots \max_{a_m} \sum_{e_m} M(e_m \mid æ_{<m} a_m) \sum_{k=1}^{\infty} \gamma^k r_k$$

- Learn any computable environment
- Acts Bayes-optimally
- One-equation theory for Artificial General Intelligence
- Computation time: exponential × infinite



$a_1 = \pi_{\text{AIXI}}(\epsilon)$

$e_1 \sim M(\cdot \mid a_1)$

$a_2 = \pi_{\text{AIXI}}(a_1 e_1)$

$e_2 \sim M(\cdot \mid a_1 e_1 a_2)$

Benefits of a Foundational Theory of AI

AIXI/UAI provides

- (High-level) blue-print or inspiration for design
- Common terminology and goal formulation
- Understand and predict behaviour of yet-to-be-built agents
- Appreciation of fundamental challenges (e.g. exploration/exploitation)
- Definition/measure of intelligence

How to approximate AIXI?

Approximating AIXI

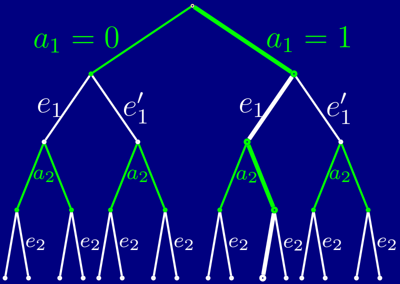
Approaches:

- MC-AIXI-CTW:
  Approximate Solomonoff induction and expectimax planning
- Feature Reinforcement Learning:
  Reduce histories to states
- Model-Free:
  Combine induction and planning

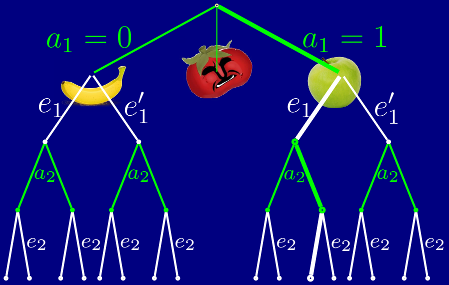
MC-AIXI-CTW: Approximating Expectimax

Planning with expectimax search takes exponential time

Sample paths in expectimax tree (anytime algorithm)



Monte Carlo Tree Search



$$a_1 = \arg\max_a V^+(a)$$

$$P(e_1 \mid a_1)$$

$$a_2 = \arg\max_a V^+(a_1 e_1 a)$$

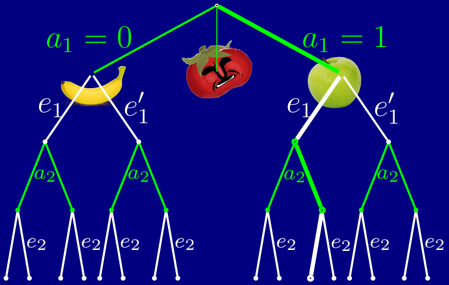
$$P(e_2 \mid a_1 e_1 a_2)$$

upper confidence bound

$$\boxed{V^+(a) = \underbrace{\hat{V}(a)}_{\text{average}} + \underbrace{\sqrt{\log T / T(a)}}_{\text{exploration bonus}}}$$

- unexplored: high $\log T/T(a)$
  $T(a) =$ times explored (a)
- promising: high $\hat{V}(a)$

Monte Carlo Tree Search



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- unexplored: high $\log T / T(a)$
  $T(a) =$ times explored (a)
- promising: high $\hat{V}(a)$

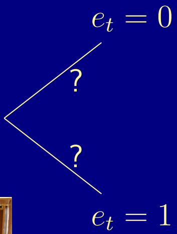
MCTS famous for good performance in Go (Gelly et al., 2006)

Approximating Solomonoff Induction

Environments $\mu(e_t \mid æ_{<t}a_t)$ allowed arbitrary long dependencies: e_{1000} may depend on a_1

Usually, most recent actions and percepts (=context) more relevant

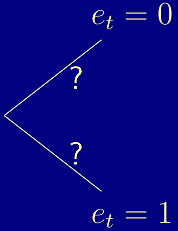
$$a_1\, e_1\, a_2\, e_2 \dots\dots\dots a_{t-3}e_{t-3}\, \underbrace{e_{t-2}\, a_{t-1}\, e_{t-1}\, a_t}_{\text{context}}$$



Learning from Contexts

The same context might have occurred before

$$\ldots 0011\mathbf{1}\ldots 0011\mathbf{0}\ldots 0011\mathbf{1}\ldots \underbrace{0}_{e_{t-2}}\underbrace{0}_{a_{t-1}}\underbrace{1}_{e_{t-1}}\underbrace{1}_{a_t}$$



$e_t = 0$

?

?

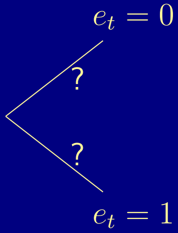
$e_t = 1$

Similar experience can be used to predict

Length of Contexts

Longer context $\Longrightarrow$ less data

$$\ldots\ldots 1000110 \ldots\ldots \underbrace{1}_{a_{t-3}} \underbrace{0}_{a_{t-2}} \underbrace{0}_{e_{t-2}} \underbrace{0}_{a_{t-1}} \underbrace{1}_{e_{t-1}} \underbrace{1}_{a_t}$$



Real-life example: I'm going to a Vietnamese restaurant tonight. Should I predict food tastiness based on previous experiences with:

- This restaurant (high precision, limited data)
- Vietnamese restaurants (medium both)
- Any restaurant (low precision, plenty of data)

Contexts – Short or Long?

Short context	More data	Less precision
Long context	Less data	Greater precision

Contexts – Short or Long?

Short context	More data	Less precision
Long context	Less data	Greater precision

Best choice depends on

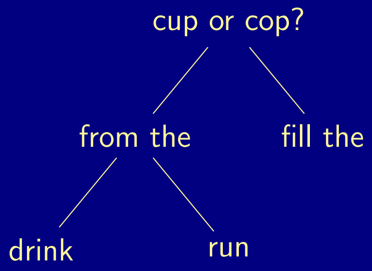
- amount of data

Contexts – Short or Long?

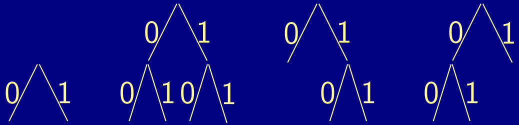
Short context	More data	Less precision
Long context	Less data	Greater precision

Best choice depends on

- amount of data
- the context itself



Context Tree Weighting (CTW)

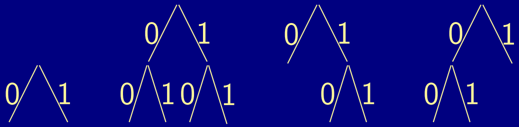


CTW "mixes" over all 2^{2^D} context trees of depth $\leq D$

$$\mathrm{CTW}(e_{<t} \mid a_{<t}) = \sum_{\Gamma} 2^{-\mathrm{CL}(\Gamma)} \Gamma(e_{<t} \mid a_{<t})$$

$$M(e_{<t} \mid a_{<t}) = \sum_{p} 2^{-\ell(p)} \, [\![\, p(a_{<t}) = e_{<t} \,]\!]$$

Context Tree Weighting (CTW)



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$$M(e_{<t} \mid a_{<t}) = \sum_{p} 2^{-\ell(p)} \, [\![\, p(a_{<t}) = e_{<t} \,]\!]$$

Computation time:

$M(e_t \mid æ_{<t} a_t)$	Infinite
$\mathrm{CTW}(e_t \mid æ_{<t} a_t)$	Constant (linear in max depth D)

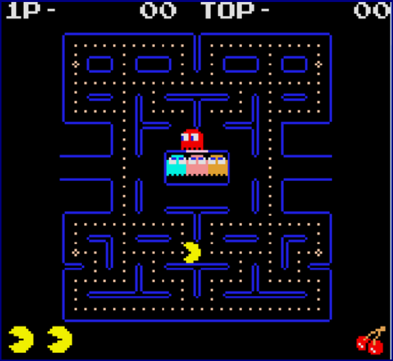
MC-AIXI-CTW

Combining Context Tree Weighting and Monte Carlo Tree Search gives MC-AIXI-CTW (Veness et al., 2011)

Learns to play

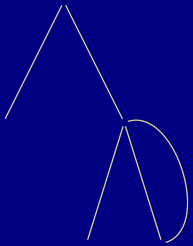
- PacMan
- TicTacToe
- Kuhn Poker
- Rock Paper Scissors

without knowing anything about the games



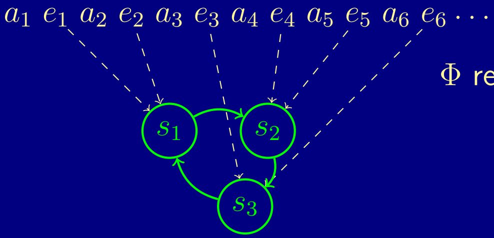
Other SI approximations

- Looping Suffix Trees (Daswani et al., 2012a)
- LSTM neural networks (Hochreiter et al., 1997)
- Speed prior (Schmidhuber, 2002; Filan et al., 2016)
- General compression techniques (Franz, AGI 2016)



Feature Reinforcement Learning

Humans generally think in terms of what state they are in.

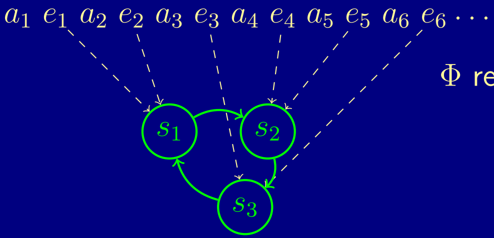


$a_1\ e_1\ a_2\ e_2\ a_3\ e_3\ a_4\ e_4\ a_5\ e_5\ a_6\ e_6 \cdots$
s_1
s_2
s_3

Φ reduces histories to states

State representation often valid:

- Games, toy problems: $\Phi(æ_{<t}) = o_t$ (state fully observable)
- Classical physics: State = position + velocity.
- General: $\Phi(æ_{<t}) = æ_{<t}$ (history is a state, but useless)


a_1 e_1 a_2 e_2 a_3 e_3 a_4 e_4 a_5 e_5 a_6 e_6 $\dots$
s_1
s_2
s_3

Φ reduces histories to states

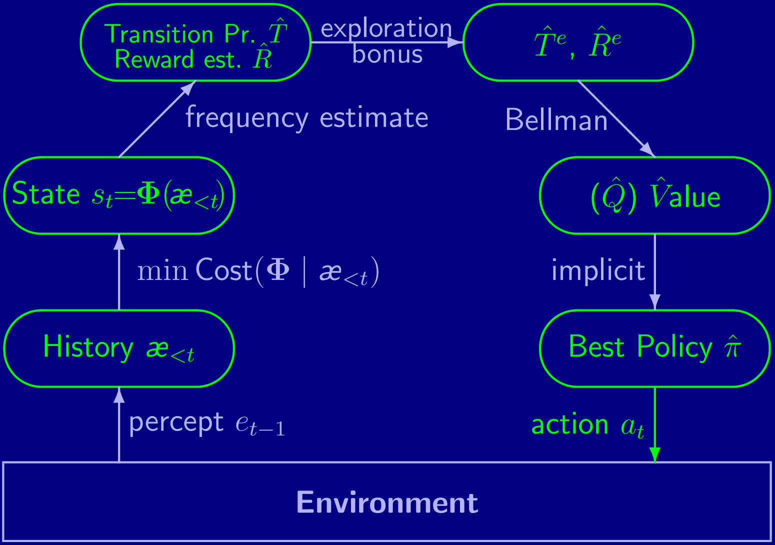
Standard RL (MDP) applications: Designers give history $\mapsto$ state

Can be inferred automatically: ΦMDP approach (Hutter, 2009b)

Search for a map $\Phi : æ_{<t} \mapsto s_i$ minimising a cost criterion

Feature Reinforcement Learning alternative to POMDPs and PSRs

ΦMDP: Computational Flow



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- Theoretical guarantees:
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- Extensions:
  - Looping context trees for long-term memory (Daswani et al., 2012b)
  - Structured MDPs (Dynamic Bayesian Networks) (Hutter, 2009a)
  - Relax Markov property (Extreme State Aggregation) (Hutter, 2014)

Model-free AIXI

Do both induction and planning simultaneously

$V^\pi(æ_{<t}a_t)$ expected return from action a_t and policy π

$V^*(æ_{<t}a_t)$ expected return from action a_t and optimal policy π^*

By learning V^*, possible to always act optimally

$$a_t = \arg\max_a V^*(æ_{<t}\, a)$$

How to learn V^* directly "Solomonoff-style" with compression is explored by Hutter (2005, Ch. 7.2) and Veness et al. (2015)

Learns ATARI games (Pong, Bass, and Q*Bert) from watching screen "DQN style"

Fundamental Challenges

- What is an optimal agent?
  - Maximum subjective reward?
  - Maximum objective reward asymptotically?
- Exploration vs. exploitation (Orseau, 2010; Leike et al., 2016a)
- Where should the reward come from?
  - Human designers
  - Knowledge-seeking agents (Orseau, 2014)
  - Utility agents (Hibbard, 2012)
  - Value learning agents (Dewey, 2011)
- How should the future be discounted? (Lattimore and Hutter, 2014)
- What is a practically feasible and general way of doing
  - induction?
  - planning?
- What is a "natural" UTM/programming language? (Mueller, 2006)
- How should agents reason about themselves? (Everitt et al., 2015)
- How should agents reason about other agents reasoning about itself? (Leike et al., 2016b)

Notions of Optimality

Should I try the new restaurant in town?

Learn whether it's good, but risk bad evening

- AIXI/Bayes-optimal:
  - Try iff higher expected utility
  - Optimal with respect to *subjective belief*
  - Any decision optimal for some belief/UTM (Leike and Hutter, 2015)
  - Subjective form of optimality
- Asymptotic optimality
  - Maximal possible reward eventually
  - Objective
  - Risky short-term

Optimism

Paradise exists, I just need to find my way there

Standard RL: Positive initialisation

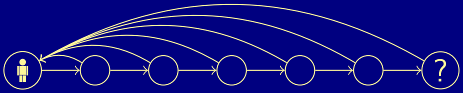
UAI: From a finite but growing set $\mathcal{N}_t$ of environments, always act according to $\nu \in \mathcal{N}_t$ that makes the highest reward possible

$$a_t^* = \arg\max_{a_t} \max_{\nu \in \mathcal{N}_t} Q_\nu(æ_{<t} a_t)$$

If there is a chance: Try it!

Optimistic agents

- explore with focus
- asymptotically optimal
  (Sunehag and Hutter, 2015)
- vulnerable to traps



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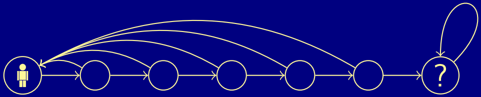

Thompson-sampling

Act according to a random environment $\nu \in \mathcal{M}$ re-sampled from posterior every effective horizon m

$$\nu \sim M(\nu \mid æ_{<t}) \quad \text{and} \quad a_t = \arg\max_a V_\nu(æ_{<t}a)$$

The more likely the restaurant is good, the higher chance try it soon. Will be tried eventually.

Thompson-sampling agents are (strongly) asymptotically optimal

(Leike et al., 2016a)

Conclusions

UAI is

- Foundational theory of AI
- What's the right thing to do

$$a_1^* = \arg\max_{a_1} \sum_{e_1} M(e_1 \mid a_1) \max_{a_2} \sum_{e_2} M(e_2 \mid a_1 e_1 a_2) \dots \max_{a_m} \sum_{e_m} M(e_m \mid æ_{<m} a_m) R(æ_{1:m})$$

$$M(e_{<t} \mid a_{<t}) = \sum_{p\,:\,p(a_{<t})=e_{<t}} 2^{-\ell(p)} \qquad R(æ_{1:\infty}) = r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots$$

Useful for

- Inspiring practical agents
- Predicting and controlling superintelligent agents
- Identifying and addressing fundamental challenges

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