Sequential Extensions of Causal and Evidential Decision Theory

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Outline

Agent Models

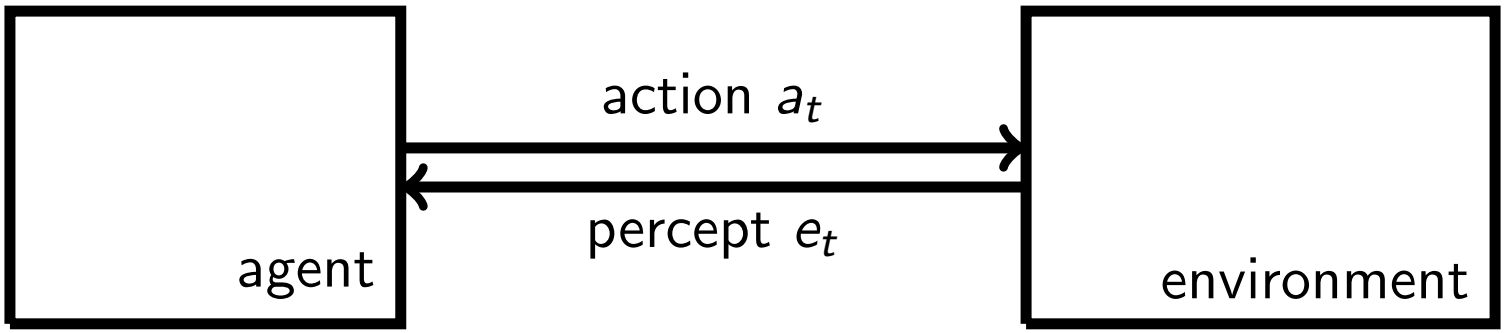
Decision Theory

Sequential Decision Making

Conclusion

References

Dualistic Agent Model



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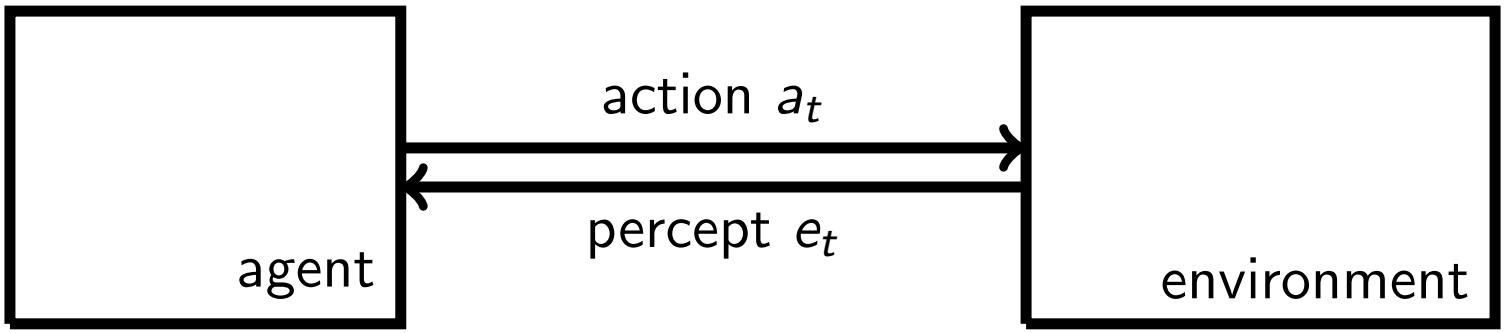
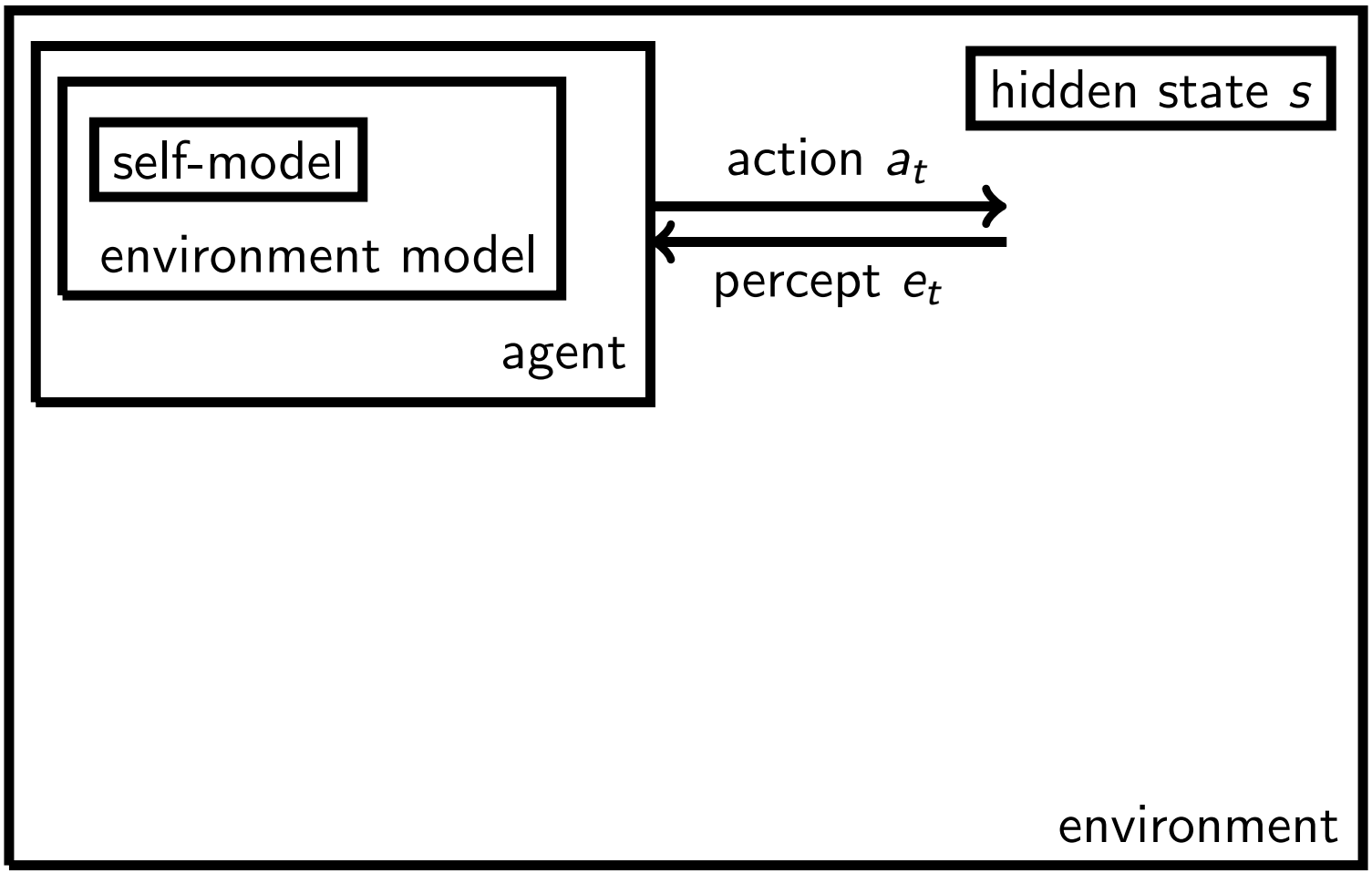


Goal: maximize expected utility $\mathbb{E}[\sum_{t=1}^{m} u(e_t)]$

Physicalistic Agent Model



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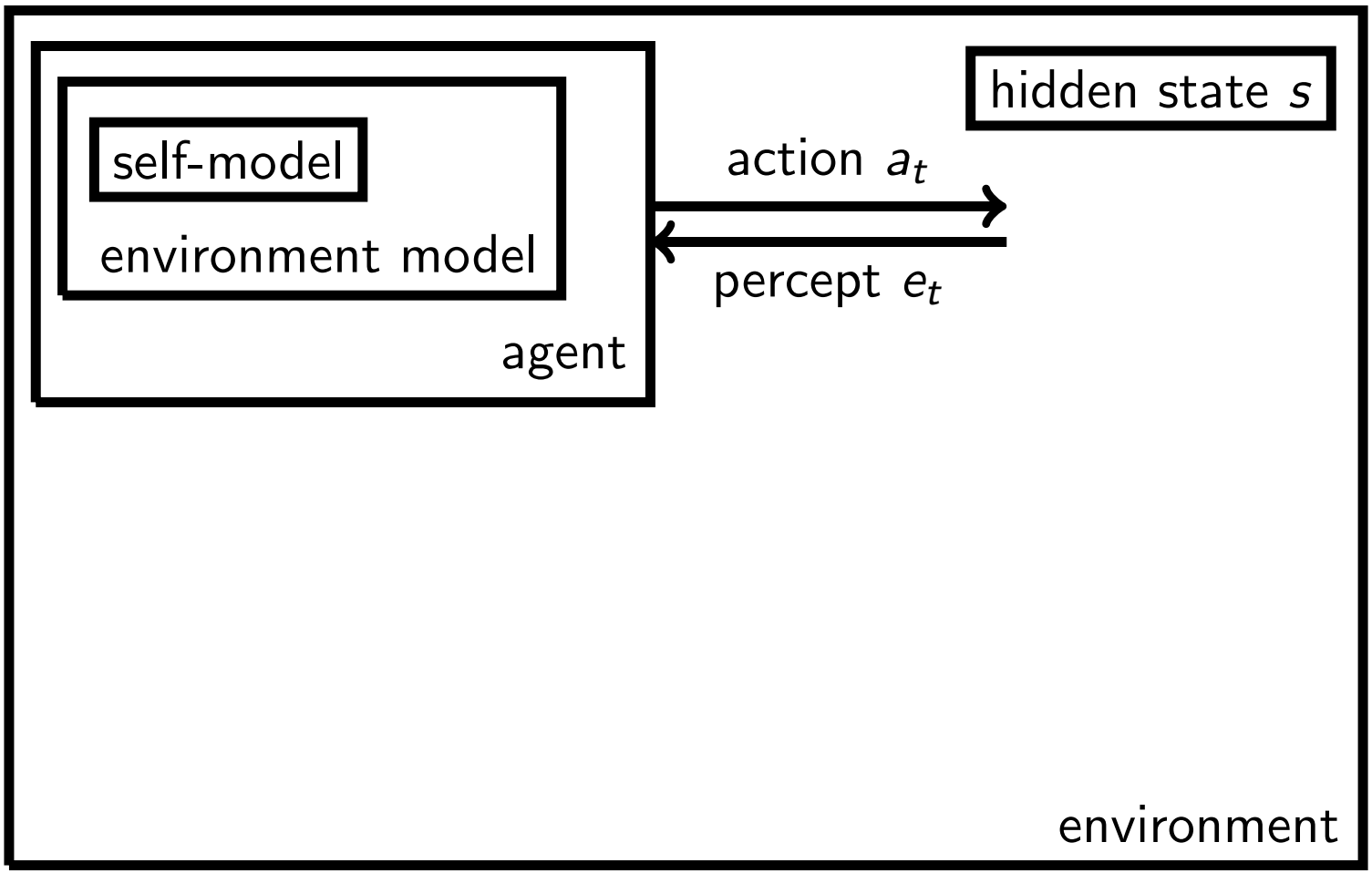

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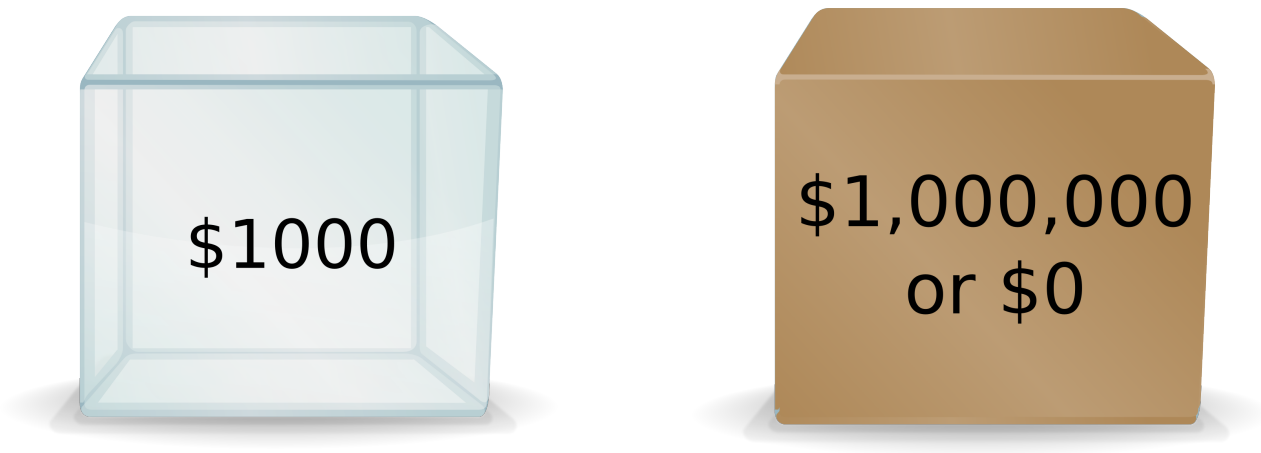
Newcomb's Problem

Presented by [Nozick, 1969]



Actions: (1) take the opaque box **or** (2) take both boxes

Reasoning Causally

Causal decision theory (CDT):

take the action that *causes* the best outcome

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$$\arg\max_{a \in \mathcal{A}} \sum_{e \in \mathcal{E}} \mu(e \mid \mathrm{do}(a))\, u(e) \qquad \text{(CDT)}$$

[Gibbard and Harper, 1978, Lewis, 1981, Skyrms, 1982, Joyce, 1999, Weirich, 2012]

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[Gibbard and Harper, 1978, Lewis, 1981, Skyrms, 1982, Joyce, 1999, Weirich, 2012]

In Newcomb's problem: taking both boxes *causes* you to have \$1000 more

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[Jeffrey, 1983, Briggs, 2014, Ahmed, 2014]

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[Jeffrey, 1983, Briggs, 2014, Ahmed, 2014]

In Newcomb's problem: taking just the opaque box is good news because that means it likely contains \$1,000,000

Newcomblike Problems

= problems where your actions are *not* independent of the (unobservable) environment state

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- Example: Environment that knows your source code
- Example: Multi-Agent setting with multiple copies of one agent

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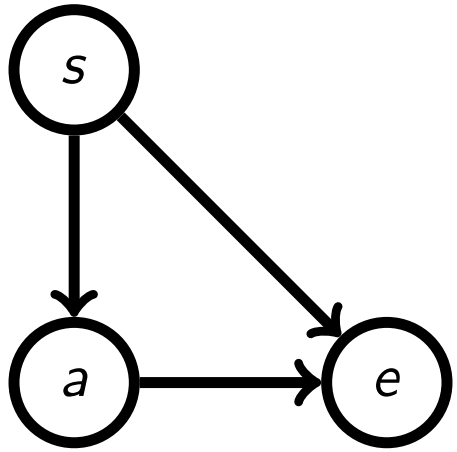
The Causal Graph

One-shot:



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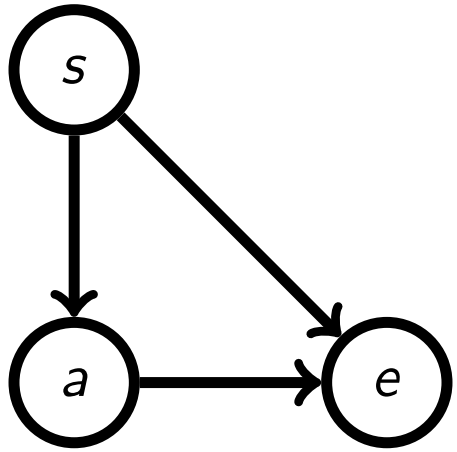
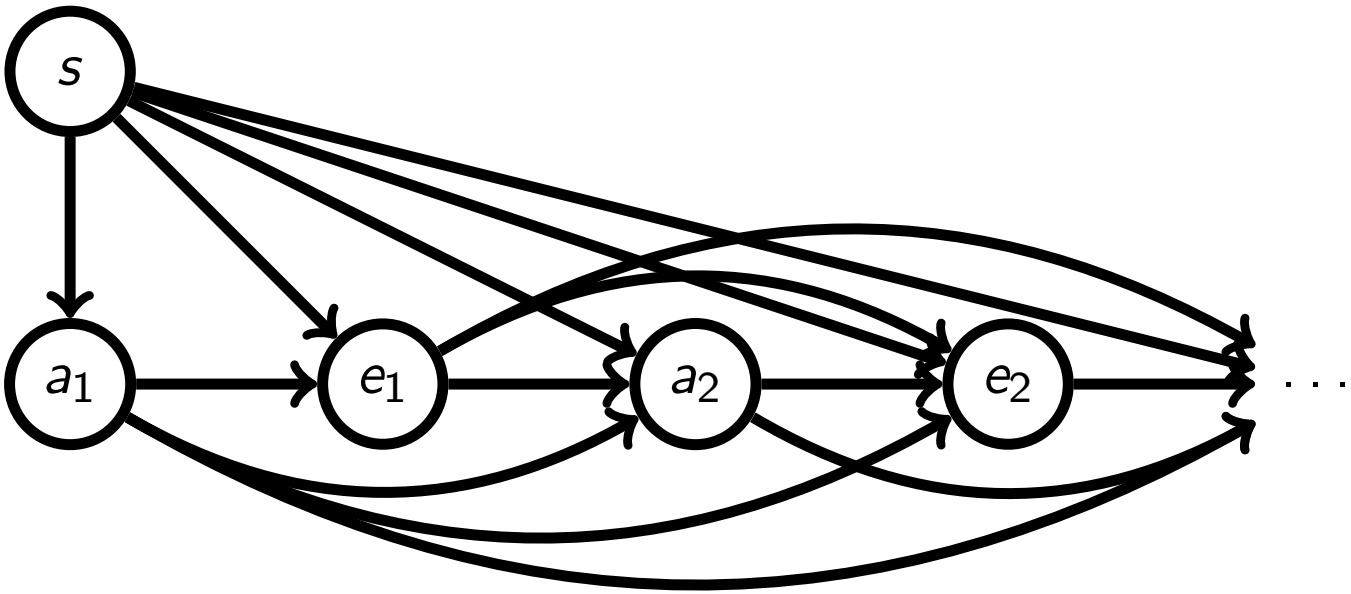


Sequential:



Notation

- $æ_{<t} = a_1 e_1 \ldots a_{t-1} e_{t-1}$ denotes the history
- $\mu : (\mathcal{A} \times \mathcal{E})^* \times \mathcal{A} \to \Delta(\mathcal{E})$ denotes the environment model
- $\pi : (\mathcal{A} \times \mathcal{E})^* \to \mathcal{A}$ is my policy
- $m \in \mathbb{N}$ is the horizon

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Sequential action-evidential decision theory (SAEDT):

$$V^{\text{aev}}(æ_{<t} a_t) := \sum_{e_t} \underbrace{\mu(e_t \mid æ_{<t} a_t)}_{\mu(e_t|\text{past},a_t)} \underbrace{\Big(u(e_t) + V^{\text{aev}}(æ_{<t} a_t e_t)\Big)}_{\text{future utility}}$$

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Sequential policy-evidential decision theory (SPEDT):

$$V^{\text{pev}}(æ_{<t} a_t) := \sum_{e_t} \underbrace{\mu(e_t \mid æ_{<t} a_t, \pi_{t+1:m})}_{\mu(e_t|\text{past},\pi)} \underbrace{\Big(u(e_t) + V^{\text{pev}}(æ_{<t} a_t e_t)\Big)}_{\text{future utility}}$$

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Sequential causal decision theory (SCDT):

$$V^{\text{cau}}(æ_{<t} a_t) := \sum_{e_t \in \mathcal{E}} \underbrace{\mu(e_t \mid æ_{<t}, \text{do}(a_t))}_{\mu(e_t | \text{past}, \text{do}(a_t))} \underbrace{\Big(u(e_t) + V^{\text{cau}}(æ_{<t} a_t e_t) \Big)}_{\text{future utility}}$$

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Proposition (Policy-Causal = Action-Causal). For all histories $æ_{<t}$ and percepts e_t: $\mu(e_t \mid æ_{<t}, \text{do}(a_t)) = \mu(e_t \mid æ_{<t}, \text{do}(\pi_{t:m}))$.

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Examples

	action-evidential	policy-evidential	causal
Newcomb	✓	✓	×
Newcomb w/ precommit	✓	✓	×
Newcomb w/ looking	×	×	×
Toxoplasmosis	×	×	✓
Seq. Toxoplasmosis	×	×	✓

Formal description in [Everitt et al., 2015] and source code at `http://jan.leike.name`

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- How physicalistic agents make decisions optimally is unsolved
- We need a better decision theory! E.g. timeless decision theory [Yudkowsky, 2010] or updateless decision theoy [Soares and Fallenstein, 2014]

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